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Final Exam, MTH 205, Summer 2009

Ayman Badawi

QUESTION 1. (**5 points**) Given $y = cos(3t)$ is a solution to the D.E $y^{(2)} + ay' + by = 0$, where a, b are constants. Find the values of a and b.

QUESTION 2. (**10 points**) Given $y = 4e^{-2x}$ is a solution to the D.E. $y^{(5)} + 2y^{(4)} - y^{(3)} - 2y^{(2)} = 0$. What is the general solution of the D.E?

QUESTION 3. (**5 points**) let

$$f(x) = \begin{cases} -x & 0 \leq x < 4 \\ 3 & x \geq 4 \end{cases}$$

Write $f(x)$ in terms of unit step functions.

QUESTION 4. (**10 points**) Solve $y' = \frac{x^3 - 2xy^3}{3y^2}$

QUESTION 5. (**10 points**) solve $y' = 2xe^{x^2-y}$

QUESTION 6. (**15 points**) Find the general solution to $y^{(2)} + \frac{1}{x}y' + \frac{1}{x^2}y = \frac{sin(ln(x))}{x^2}$

QUESTION 7. (10 points) Solve : $y(x) = \int_0^x (x-r)e^{x-r}cos(r)\ dr + \int_0^x sin(r)y(x-r)\ dr$

QUESTION 8. (10 points) Find $x(t)$ and $y(t)$, given $x(0) = x'(0) = 0$ and $y(0) = 1$
$x^{(2)}(t) + y(t) = 3$
$x'(t) + y'(t) = 2t$

QUESTION 9. (**15 points**) Given $f(x)$ is PERIODIC and is defined on $[0, \infty)$. The first period of $f(x)$ is determined by

$$\begin{cases} 1 & 0 \leq x < 1 \\ 0 & 1 \leq x < 2 \end{cases}$$

Find y(x) so that $\int_0^x f(r)\, dr + \int_0^x f(x-r)y(r)\, dr = xe^x$ [note $(1 - e^{-2s}) = (1 + e^{-s})(1 - e^{-s})$]

QUESTION 10. (**10 points**) Let $T(t)$ be the temperature of a thermometer at any time t. If a thermometer reading 34^c is placed in an oven preheated to a constant temperature T_0. Given that the rate of change of the thermometer temperature is exactly $0.2(T(t) - T_0)$. Through a glass of a window in the oven, an observer records that the thermometer reads 44^c after one minute. How hot is the oven? (i.e. Find T_0).

Faculty information

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